Synopsis

on

ON SOME MATHEMATICAL MODELS IN NEUTROSOPHIC FUZZY SYSTEM AND FRACTIONAL CALCULUS

by

PRAMODH BHARATI

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Dr. Subhabrata Mondal

and co-supervision of
Dr. Banamali Roy

DEPARTMENT OF MATHEMATICS
SCHOOL OF BASIC SCIENCES
SWAMI VIVEKANANDA UNIVERSITY
BARRACKPORE, KOLKATA- 700121, WEST BENGAL, INDIA

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SYNOPSIS

The word "model" is derived from its Latin word modulus whose meaning is 'measure'. When we use the word "model," we mean an object that is a representation of something that is real with a different scale. Discussing the definition of mathematical modelling is highly pertinent in this context. The use of mathematics to explain real-world issues and discuss the significant questions that raise is known as mathematical modelling.

Mathematical Modelling [1] is the application of mathematics to describe real-world problems and analyze the important questions that arise from it. By the help of various types of mathematical tools, the real world problem can be formed to a mathematical problem. A mathematical model consists of mathematical expressions or equations demonstrating a process or a system. It is an idealized mathematical representation of a real-world occurrence. In the practice of mathematical modeling, modelers have few objectives to observe the realistic phenomena when a good model is formulated. Using mathematical modelling, interpretation and future predictions of some facts can be addressed. Real world challenges include issues from a variety of academic fields, including biology, bioeconomics, agriculture, social sciences, engineering, physics, chemistry, ecology, environment, and business and finance. The flow diagram of mathematical modelling is given below:

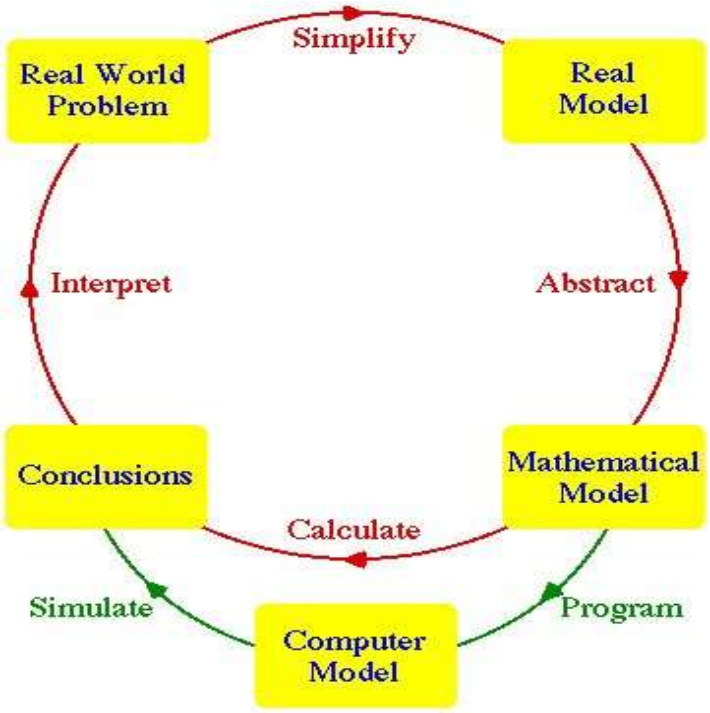

Figure 1: Schematic diagram of mathematical modelling

Operations Research (OR) [2–6] is an intriguing area that includes many branches such as op-

timization, linear programming, dynamic programming, nonlinear programming, supply chain management, simulation, inventory theory, queueing theory, graph Graph Theory, etc. to address challenging issues across a range of industries. The core concept of operations research is mathematical modeling. It is the process of employing mathematical structures to represent elements of the real world. These models can be defined in a variety of ways in applicable problems.

In the last few decades, mathematical modelling has been increasingly important in the biological sciences. Research on the ecological dynamics of biological systems through mathematical modelling is becoming more and more popular. In order to examine the modelling of biological phenomena, we must concentrate on a few fundamental processes. These consist of interpretation, analysis, and formulation. Additionally Innovative and technical methods are required for both the model's illustration and the development of its implications in biological modelling. The majority of biological models are created using differential equations, including both ODE and PDE kinds, or difference equations because when the state variable or space is altered by various time series, the model's behavior is altered as well.

The concept of fractional differential equations emerges from contemporary mathematics and is among various abstract mathematical notions [23]. Systems involving fractional orders encompass the notion of temporal memory, a feature absent in systems with integer orders. In contrast to the integer-order derivative, which reveals changes or particular qualities at different time occurrences, the fractional-order derivative represents the whole temporal range of a biological process [24]. Despite the paucity of current research on fractional-order delays [25–27], we are now concentrating on building a system with both delays and fractional-order. This method offers a fresh viewpoint on interactions between predators and prey.

A sensitivity heat map was utilized in a different study [28] to apply the Caputo fractional derivative in a gonorrhea transmission model within a sex-structured population. In a related investigation, Asamoah et al. [29] used a fractional derivative and a suggested mathematical model to investigate heartwater, a disease that infects ruminants and is carried by ticks. Fractional-order models can provide a significant historical perspective on illnesses in epidemiological models, which helps to improve understanding of complex disease dynamics [30]. The study of infectious disease behavior has benefited greatly from the use of mathematical models [31]]. A fractional-order model of the smoking pandemic that was age-specific was presented by Addai et al. [32]. Note the following studies on the impact of fractional derivative on monkeypox illness [33]. It was vividly noticed that the memory effect demonstrated in the dynamics of the disease has a major influence on the length of time it may take for a solution trajectory to be in a steady state. This dissertation addresses some mathematical models utilizing the interval differential equation idea and neutrosophic set theory. In addition, we apply the concept of fractional calculus to investigate the fractional order prey-predator model.

We have divided our entire proposed thesis into the following divisions and chapters:

Part-A: Introduction, basic concepts and preliminaries

Chapter 1: Introduction
Chapter 2: Basic concepts

Part-B: Biomathematical modelling under interval and intuistionistic environment

Chapter 3: Studies of dynamical behaviors of tri-tropic food chain model with two fear effects under interval uncertainty
3.1: Analysis of tri-tropic food chain model with two fear effects under interval uncertainty
3.2: Results and discussion tri-tropic food chain model with two fear effects and harvesting top and middle predator under interval uncertainty
Chapter 4: Study of two compartment drug concentration model using the concept of intuitionistic fuzzy sets

Part-C: Mathematical Modelling of optimization problems under neutrosophic environment

Chapter 5: Ranking of neutrosophic number based on values and ambiguities and its application to linear programming problem
Chapter 6: An attempt on neutrosophic assignment problem based on weighted magnitude method
Chapter 7: A neutrosophic inventory model using trapezoidal neutrosophic number

Part-D: Prey-predator model using fractional calculus

Chapter 8: Effect of fear in a fractional order prey predator model with time delayed carring capacity

Part-E: Summary of the thesis and futrure research scopes

Chapter 9: Overall discussion and future research scopes

In **Chapter 1**, a brief introduction that consists of historical background, mathematical preliminaries and motivation about research works have been described.

In **Chapter 2**, the definition of fuzzy sets, interval number and their few properties, Existence and uniqueness theorem of differential equation with using interval- valued function and the definitions of Intuitionistic fuzzy set(IFS) and their properties have been stated.

Chapter 3 studies of dynamical behaviours of tri-tropic food chain model with two fear effects under interval uncertainty. Now a days mathematics and biology have a synergistic relationship. The study of prey-predator dynamics is crucial part of the ecology. But parameters present into the ecological model are fixed due to several reasons like demographic fuzziness, environmental reason etc. In this chapter, we consider the tri-trophic food chain model where middle predator is generalised and top predator is specialist.

Model 1: The density of the prey , intermediate predator, and top predator population, respectively, at any given time t be represented by the variables $u(t), v(t)$ and $w(t)$ .The following presumption is made in order to formulate the model system: $u(t)$and$v(t)$increases at growth rate of$r_1$andr_2, with two carrying capacityK_1 and K_2, p_1 and p_2 represents the predation rate, according to Holling Type-II functional response $q_1$and $q_2$represents the rate of conversion, d_1 represent natural date rate,m_1 and m_2 represents the fear levels,c_1 represent half saturation constant. Our proposed bio-mathematical model is

$$\begin{aligned}
\frac{du(t)}{dt} &= \frac{r_1 u}{1+m_1 w}\left(1-\frac{u}{K_1}\right)-p_1 uv,\\
\frac{dv(t)}{dt} &= \frac{r_2 v}{1+m_2 w}\left(1-\frac{v}{K_2}\right)+p_2 uv-\frac{q_1 vw}{c_1+v}.\\
\frac{dw(t)}{dt} &= \frac{q_2 vw}{c_1+v}-d_1 w,
\end{aligned} \tag{1}$$

Model in imprecise environment

The proposed model (1) in the imprecise environment can be changed the coefficients taken as interval number as follows:

$$\begin{aligned}
\frac{du(t)}{dt} &= \frac{\widehat{r_1} u}{1+\widehat{m_1} w}\left(1-\frac{u}{K_1}\right)-\widehat{p_1} uv,\\
\frac{dv(t)}{dt} &= \frac{\widehat{r_2} v}{1+\widehat{m_2} w}\left(1-\frac{v}{K_2}\right)+\widehat{p_2} uv-\frac{\widehat{q_1} vw}{\widehat{c_1}+v}.\\
\frac{dw(t)}{dt} &= \frac{\widehat{q_2} vw}{\widehat{c_1}+v}-\widehat{d_1} w,
\end{aligned} \tag{2}$$

Now, we take $I_l(\rho) = I_{1L}^{(1-\rho)} I_{1R}^{\rho}$ for$\rho \in [0,1]$for an interval $[I_{1L}, I_{1R}]$. Then the above system (2) can

write as follows:

$$\begin{cases} \frac{du}{dt} = \frac{r_{1L}^{1-\rho} r_{1R}^{\rho}}{1+wm_{1L}^{1-\rho} m_{1R}^{\rho}} u\left(1-\frac{u}{k_1}\right) - p_{1R}^{1-\rho} p_{1L}^{\rho} uv, \\ \frac{dv}{dt} = \frac{r_{2L}^{1-\rho} r_{2R}^{\rho}}{1+wm_{2L}^{1-\rho} m_{2R}^{\rho}} u\left(1-\frac{v}{k_2}\right) + p_{2L}^{1-\rho} p_{2R}^{\rho} uv - \frac{q_{1R}^{1-\rho} q_{1L}^{\rho}}{c_{1R}^{1-\rho} c_{1L}^{\rho}+v} vw, \\ \frac{dw}{dt} = \frac{q_{2L}^{1-\rho} q_{2R}^{\rho}}{c_{1R}^{1-\rho} c_{1L}^{\rho}+v} vw - d_{1R}^{1-\rho} d_{1L}^{\rho} w. \end{cases} \tag{3}$$

where $\rho \in [0,1]$.

Model 2

Model Formulation: Let $u(t), v(t)$ and $w(t)$ represent the density of the prey population, intermediate predator, and top predator, respectively, at any time t. To formulate the model system, the following assumption is made: $u(t)$ and $v(t)$ increases at growth rate of $r_1$and r_2, with two carrying capacity K_1 and K_2,p_1 and p_2 represents the predation rate, according to Holling Type-II functional response q_1 andq_2 represents the rate of conversion, p_3 represent natural date rate, m_1 and m_2 represents the fear levels, c_1 represent half saturation constant. Let E_1 and E_2 be the harvesting effort and d_1, d_2be the catchability coefficients of intermediate predator and top predator respectively. Our proposed bio-mathematical model is

$$\begin{aligned} \frac{du(t)}{dt} &= \frac{r_1 u}{1+m_1 w}\left(1-\frac{u}{K_1}\right) - p_1 uv, \\ \frac{dv(t)}{dt} &= \frac{r_2 v}{1+m_2 w}\left(1-\frac{v}{K_2}\right) + p_2 uv - \frac{q_1 vw}{c_1+v} - E_1 d_1 v. \\ \frac{dw(t)}{dt} &= \frac{q_2 vw}{c_1+v} - p_1 w - E_2 d_2 w, \end{aligned} \tag{4}$$

Model in imprecise environment
The proposed model (4) in the imprecise environment can be changed the coefficients taken as interval number as follows:

$$\begin{aligned} \frac{du(t)}{dt} &= \frac{\widehat{r_1} u}{1+\widehat{m_1} w}\left(1-\frac{u}{K_1}\right) - \widehat{p_1} uv, \\ \frac{dv(t)}{dt} &= \frac{\widehat{r_2} v}{1+\widehat{m_2} w}\left(1-\frac{v}{K_2}\right) + \widehat{p_2} uv - \frac{\widehat{q_1} vw}{\widehat{c_1}+v} - E_1 d_1 v. \\ \frac{dw(t)}{dt} &= \frac{\widehat{q_2} vw}{\widehat{c_1}+v} - \widehat{p_1} w - E_2 d_2 w, \end{aligned} \tag{5}$$

Now, we take $I_l(\rho_1) = I_{1L}^{(1-\rho_1)} I_{1R}^{\rho}$ for$\rho_1 \in [0,1]$for an interval $[I_{1L}, I_{1R}]$. Then the above system (5) can write as follows:

$$\begin{cases} \frac{du}{dt} = \frac{r_{1L}^{1-\rho_1} r_{1R}^{\rho_1}}{1+wm_{1L}^{1-\rho_1} m_{1R}^{\rho_1}} u\left(1-\frac{u}{k_1}\right) - p_{1R}^{1-\rho_1} p_{1L}^{\rho_1} uv, \\ \frac{dv}{dt} = \frac{r_{2L}^{1-\rho_1} r_{2R}^{\rho_1}}{1+wm_{2L}^{1-\rho_1} m_{2R}^{\rho_1}} u\left(1-\frac{v}{k_2}\right) + p_{2L}^{1-\rho_1} p_{2R}^{\rho_1} uv - \frac{q_{1R}^{1-\rho_1} q_{1L}^{\rho_1}}{c_{1R}^{1-\rho_1} c_{1L}^{\rho_1}+v} vw - E_1 d_1 v, \\ \frac{dw}{dt} = \frac{q_{2L}^{1-\rho_1} q_{2R}^{\rho_1}}{c_{1R}^{1-\rho_1} c_{1L}^{\rho_1}+v} vw - d_{1R}^{1-\rho_1} d_{1L}^{\rho_1} w - E_2 d_2 w. \end{cases} \tag{6}$$

where $\rho \in [0,1]$.

In **Chapter 4** ,we examine the application of a system of ordinary differential equations by developing a two-compartment drug concentration model utilizing intuitionistic fuzzy set theory. This chapter employs generalized Hukuhara derivatives for intuitionistic fuzzy-valued functions as a solution technique for addressing the intuitionistic drug concentration problem..

In **Chapter 5**,we establish a methodology for ranking of single-valued neutrosophic numbers on the basis of values and ambiguities.. Applying this ranking method, neutrosophic linear programming problem converts to the crisp linear programmning problems and solved numerically. At last, for the stability of the ranking function, some examples are provided for comparing this method with other approaches .

In **Chapter 6** , we develop magnitude method to find magnitude of neutrosophic number and using this method we solved assignment problem in neutrosophic environment. To justify the efficiency of the proposed model, we give a daily life example and the example is explained numerically and solved.

In **Chapter 7**,we establish a methodology for ranking of single-valued neutrosophic numbers on the basis of values and ambiguities.. Applying this ranking method, neutrosophic linear programming problem converts to the crisp linear programmning problems and solved numerically. At last, for the stability of the ranking function, some examples are provided for comparing this method with other approaches. .

In **Chapter 8**, the model formulation with assumption is given bellow

Step 1: Interactions between predators and prey are commonly observed in ecological systems. Employing mathematical models is advantageous in elucidating their behaviors. Let $u(t)$ and $v(t)$ denote the prey and predator densities at a given time t. In constructing the model system, the subsequent assumption is considered:

(a) When there is no predator present, the prey population $u(t)$ grows following a logistic pattern

with an intrinsic growth rate denoted as r and a carrying capacity represented by K.

(b) The predator population $v(t)$ feeds on the prey population $u(t)$ using a Holling Type-II functional response, with a consumption rate of a_1 and a half-saturation constant denoted as h_1 for the prey population $u(t)$.
Thus we can write

$$\frac{du(t)}{dt} = ru(t)\left(1 - \frac{u(t)}{K}\right) - \frac{a_1 u(t)v(t)}{h_1 + u(t)}.$$

Consider a_2 as the efficiency of converting prey into predator biomass, and let d_1 represent the mortality rate of the predator population $v(t)$.
Then we have

$$\frac{dv(t)}{dt} = \frac{a_2 u(t)v(t)}{h_1 + u(t)} - d_1 v(t).$$

Considering all the above assumption, we have

$$\begin{aligned} \frac{du(t)}{dt} &= ru(t)\left(1 - \frac{u(t)}{K}\right) - \frac{a_1 u(t)v(t)}{h_1 + u(t)}, \\ \frac{dv(t)}{dt} &= \frac{a_2 u(t)v(t)}{h_1 + u(t)} - d_1 v(t). \end{aligned} \tag{7}$$

Step 2: A widely accepted notion is that persistent fears triggered by predators are believed to impact the fundamental reproduction of the prey population. Subsequently, we formulate the model incorporating the factor of fear $f(\sigma_1, v) = \frac{1}{(1+\sigma_1 v(t))}$, where σ_1 signifies the extent of predator-induced fear on the prey species. This can be expressed as:

$$\begin{aligned} \frac{du(t)}{dt} &= \frac{r}{(1 + \sigma_1 v(t))}u(t)\left(1 - \frac{u(t)}{K}\right) - \frac{a_1 u(t)v(t)}{h_1 + u(t)}, \\ \frac{dv(t)}{dt} &= \frac{a_2 u(t)v(t)}{h_1 + u(t)} - d_1 v(t). \end{aligned} \tag{8}$$

The function $f(\sigma_1, v) = \frac{1}{(1+\sigma_1 v(t))}$ is referred to as the fear function. In the context of biological implications involving σ_1, v, and $f(\sigma_1, v)$, it is reasonable to make the following assumption:

$f(0, v) = 1, f(\sigma_1, 0) = 1, \lim_{\sigma_1 \to \infty} f(\sigma_1, v) = 0, \lim_{v \to \infty} f(\sigma_1, v) = 0, \frac{\partial f(\sigma_1, v)}{\partial \sigma_1} < 0, \frac{\partial f(\sigma_1, v)}{\partial v} < 0.$

Step 3: The carrying capacity of a species plays a vital role in determining the maximum population size achievable based on factors like environmental situations and available resources such as food, water, and habitat. However, the growth of a species can be notably influenced by changing

environmental factors, leading to fluctuations in its size. This makes carrying capacity variable and subject to change over time, introducing an element of uncertainty that enhances ecological realism in dynamics [7]. Yukalov et al. [8,9] presented a concept of delay-dependent carrying capacity, considering a species' influence on its intrinsic carrying capacity. Rather than a fixed value, the carrying capacity becomes a function of past population size. This implies that conservation of resources can bolster the carrying capacity, while resource exploitation can reduce it. Importantly, the carrying capacity shaped by a species' activities doesn't manifest immediately; it's subject to a delay. Consequently, the induced carrying capacity is influenced by the population size over a certain time span. Based on this idea, Yukalov et al. [8,9] introduced the subsequent carrying capacity formulation:

$$K = K_0 + \alpha N(t-\tau). \tag{9}$$

Here, K_0 stands for the initial (natural, inherent) carrying capacity, and $\alpha N(t-\tau)$ signifies the induced carrying capacity resulting from past activities of the species (N) itself. The parameter α in the real numbers gauges the influence of these activities on the carrying capacity. Moreover, $\tau > 0$ represents a time lag, depicting the effect of past population size on the present carrying capacity. Using this functional carrying capacity, Yukalov et al. [8,9] explored the dynamics of a logistic model for a single species. They identified phenomena like punctuated evolution, the presence of Hopf bifurcation, and oscillatory patterns with finite-time extinction, all attributed to the time lag. Guided by this motivation, the present study focuses on investigating the dynamics of the predator-prey model (8), while considering the delay-dependent carrying capacity (9) for the prey population.
Hence, the formulation of our predator-prey system can be expressed as follows:

$$\begin{aligned}
\frac{du(t)}{dt} &= \frac{r}{(1+\sigma_1 v(t))}u(t)\left(1-\frac{u(t)}{K_0+\alpha u(t-\tau)}\right)-\frac{a_1 u(t)v(t)}{h_1+u(t)},\\
\frac{dv(t)}{dt} &= \frac{a_2 u(t)v(t)}{h_1+u(t)}-d_1 v(t).
\end{aligned} \tag{10}$$

Step 4: We have examined the proposed model using the Caputo derivative of order $0 < \phi \leq 1$.

$$\begin{aligned}
{}^{C}D_t^{\phi}u(t) &= \frac{\tilde{r}^{\phi}}{(1+\tilde{\sigma_1}^{\phi}v(t))}u(t)\left(1-\frac{u(t)}{\tilde{K_0}^{\phi}+\tilde{\alpha}^{\phi}u(t-\tau)}\right)-\frac{\tilde{a_1}^{\phi}u(t)v(t)}{\tilde{h_1}^{\phi}+u(t)},\\
{}^{C}D_t^{\phi}v(t) &= \frac{\tilde{a_2}^{\phi}u(t)v(t)}{\tilde{h_1}^{\phi}+u(t)}-\tilde{d_1}^{\phi}v(t).
\end{aligned} \tag{11}$$

Next, we reconfigure the parameters for the purpose of facilitating computational convenience: $\tilde{r}^{\phi} = r, \tilde{\sigma_1}^{\phi} = \sigma_1, \tilde{K_0}^{\phi} = K_0, \tilde{\alpha}^{\phi} = \alpha, \tilde{a_1}^{\phi} = a_1, \tilde{a_2}^{\phi} = a_2, \tilde{h_1}^{\phi} = h_1, \tilde{d_1}^{\phi} = d_1$.

Starting from initial conditions, the enhanced model system (5) can ultimately be formulated as follows:

$$\begin{aligned} {}^{C}D_{t}^{\phi}u(t) &= \frac{r}{(1+\sigma_1 v(t))}u(t)\left(1-\frac{u(t)}{K_0+\alpha u(t-\tau)}\right)-\frac{a_1 u(t)v(t)}{h_1+u(t)}, \\ {}^{C}D_{t}^{\phi}v(t) &= \frac{a_2 u(t)v(t)}{h_1+u(t)}-d_1 v(t), \end{aligned} \tag{12}$$

with $u(0) > 0, v(0) > 0$.

We summarize the overall thesis work and suggest few future work directions of our studies in **Chapter 9**

Bibliography

[1] Banerjee,S., Mathematical Modeling: Models, Analysis and Applications (2nd ed.). Chapman and Hall/CRC, https://doi.org/10.1201/9781351022941

[2] Karmakar S., De S.K, Goswami A., A polution sensitive remanufacturing model with waste item: Triangular dense fuzzy lock set approah, Journel of cleaner production, 187(2018), 789-803.

[3] T.Bera and N.K.Mahapatra (2019),Assignment problem with neutrosophic costs and its solution methodology, Asia Mathematika, 3(1), 21-32.

[4] Bera, T., Mahapatra, N.K. (2019) Generalised Single Valued Neutrosophic Number and Its Application to Neutrosophic Linear Programming, Neutrosophic Sets and Systems, 25 (1).

[5] Karak M, Mahata. A, Rong. M, Mukherjee. S, Mondal S.P, Broumi. S, Roy. B (2023), A solution technique of Transportation Problem in Neutrosophic Environment, Neutrosophic System with applications, Vol.3.

[6] A.N. Revathi, S. Mohanaselvi, and Broumi Said (2023), An Efficient Neutrosophic Technique for Uncertain Multi Objective Transportation Problem, Neutrosophic Sets and Systems, vol.53.

[7] Liang S, Wu R, Chen L. Laplace transform of fractional order differential equations. Electronic Journal of Differential Equations 2015;24(12):2019-2023. doi.org/10.1016/j.aml.2011.05.035.

[8] Ruan S, Ardito A, Ricciardi P, DeAngelis DL. Coexistence in competition models with density-dependent mortality. Comptes rendus biologies 2007;330:845–854. DOI: 10.1016/j.crvi.2007.10.004.

[9] Sabatier J, Agrawal OP, Tenreiro Machado JA. Advances in Fractional Calculus: Theoretical Developments and Applications in Physics and Engineering, Springer 2007.

[10] Jana S, Chakraborty M, Chakraborty K, Kar TK., Global stability and bifurcation of time delayed prey–predator system incorporating prey refuge. Math Comput Simul 2012;85:57–77.

[11] Dubey B, Kumar A., Dynamics of prey–predator model with stage structure in prey including maturation and gestation delays. Nonlinear Dynam 2019;96:2653–2679.

[12] Dubey B, Sajan, Kumar A., Stability switching and chaos in a multiple delayed prey–predator model with fear effect and anti-predator behavior. Math Comput Simul 2021;188:164–192.

[13] Matsumoto A, Szidarovszky F., Stability switching curves in a Lotka–Volterra competition system with two delays. Math Comput Simul 2020;178:422–438.

[14] Del Monte-Luna P, Brook BW, Zetina-Rejon MJ, Cruz-Escalona VH. The carrying capacity of ecosystems. Glob Ecol Biogeogr 2004;13:485–495.

[15] Naveed M, Raza A, Soori AH, et al. A dynamical study of diarrhea delayed epidemic model: application of mathematical biology in infectious diseases. Eur Phys J Plus 2023;138:664.

[16] Yukalov VI, Yukalova EP, Sornette D. Extreme events in population dynamics with functional carrying capacity. Eur Phys J Spec Top 2012;205:313–354.

[17] Barman D, Naskar S, Mandal A., et al. Impact of seasonal variability of sea waves on the dynamics of a predator–prey system. Eur Phys J Plus 223;138:641.

[18] Layek GC. An Introduction To Dynamical Systems And Chaos. Springer 2015.

[19] Celik C. The stability and hopf bifurcation for a predator–prey system with time delay. Chaos Solitons Fractals 2008;37:87–99.

[20] Agrawal R, Jana D, Upadhyay RK, Rao V.S.H., Complex dynamics of sexually reproductive generalist predator and gestation delay in a food chain model: double Hopf-bifurcation to chaos. J Appl Math Comput 2017;55:513–547.

[21] Wang J, Jiang W., Hopf-zero bifurcation of a delayed predator–prey model with dormancy of predators. J Appl Anal Comput 2017;7:1051–1069.

[22] Banerjee J, Sasmal S.K, Layek R.K., Supercritical and subcritical Hopf-bifurcations in a two-delayed prey–predator system with density-dependent mortality of predator and strong Allee effect in prey. Biosystems 219;180:19–37.

[23] Podlubny I. Fractional Differential Equations. Academic Press, San Diego 1999.

[24] Huang C, Song X, Fang B, Xiao M, Cao J. Modeling, Analysis and Bifurcation Control of a Delayed Fractional-Order Predator–Prey Model. International Journal of Bifurcation and Chaos 2018;28(9):1850117.

[25] Rihan FA, Lakshmanan S, Hashish AH, Rakkiyappan R, Ahmed E. Fractional-order delayed predator-prey systems with Holling type-II functional response. Nonlinear Dyn 2015;80:777–789.

[26] Song P, Zhao H, Zhang X. Dynamic analysis of a fractional order delayed predator-prey system with harvesting. Theory Biosci 2016;135, 59–72.

[27] Zhang H, Cai Y, Fu S, Wang W. Impact of the fear effect in a prey-predator model incorporating a prey refuge. Applied Mathematics and Computation 2019; 356:328-337. https://doi.org/10.1016/j.amc.2019.03.034.

[28] Asamoah J.K.K., Sun GQ. Fractional Caputo and sensitivity heat map for a gonorrhea transmission model in a sex structured population. Chaos Solitons Fractals 2023;114026. http://dx.doi.org/10.1016/j.chaos.2023.114026.

[29] Asamoah J.K.K., Fatmawati: A fractional mathematical model of heartwater transmission dynamics considering nymph and adult amblyomma ticks. Chaos Solitons Fractals 2023;174:113905. http://dx.doi.org/10.1016/j.chaos.2023.113905.

[30] Asamoah J.K.K., Okyere E, Yankson E, Opoku AA, Konadu AA, Acheampong E, et al. Non-fractional and fractional mathematical analysis and simulations for Q fever. Chaos Solitons Fractals 2022;111821. http://dx.doi.org/10.1016/j.chaos.2022.111821.

[31] Addai E, Zhang L, Preko AK, Asamoah JKK. Fractional order epidemiological model of SARS-CoV-2 dynamism involving Alzheimer's disease, Healthc. Anal. 2022;2,100114.

[32] Addai E, Zhang L, Asamoah JK, Essel JF. A fractional order age-specific smoke epidemic model. Appl Math Model 2023;119:99–118.

[33] Adom-Konadu A, Bonyah E, Sackitey AL, Anokye M, Asamoah JKK. A fractional order monkeypox model with protected travelers using the fixed point theorem and Newton polynomial interpolation, Healthc. Anal. 2023;3:11. http://dx.doi.org/10.1016/j.health.2023.100191.